

TABLE I
SELECTIONS OF SYMBOLS ($m \neq 0$) FOR EIGHT CASES IN FIG. 2

Symbol	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
Σ	Σ'	Σ''	Σ''	Σ'	Σ'	Σ'	Σ''	Σ'
γ_m	$m\pi/a$	$m\pi/2a$	$m\pi/2a$	$m\pi/a$	$m\pi/a$	$m\pi/a$	$m\pi/2a$	$m\pi/a$
A_m	cos	sin	sin	cos	sin	sin	sin	sin
D_m	MK_1	MK_2	MK_1	MK_1	MK_2	MK_1	MK_1	MK_1
$MK_1(n)$	cosh	cosh	cosh	sinh	cosh	cosh	sinh	sinh
$MK_2(n)$	sinh	sinh	sinh	cosh	sinh	sinh	cosh	cosh
$YK_1(1)$	cosh	sinh	cosh	cosh	sinh	cosh	cosh	cosh
$YK_2(1)$	sinh	cosh	sinh	sinh	cosh	sinh	sinh	sinh

$$\Sigma' = \sum_{m=1,2,3}^{\infty}; \quad \Sigma'' = \sum_{m=1,3,5}^{\infty}; \quad MK(n) \equiv MK(\epsilon_n^*, \epsilon_{n+1}^*, d_n, d_{n-1}, \gamma_m);$$

$$YK(1) \equiv YK(\epsilon_0^*, \epsilon_1^*, d_1, d_0, \gamma_m).$$

$$\begin{aligned} &MK_3(\epsilon_i^*, \epsilon_{i+1}^*, d_i, d_{i-1}) \\ &\equiv \epsilon_i^* MK_3(\epsilon_{i+1}^*, \epsilon_{i+2}^*, d_{i+1}, d_i) \\ &\quad + \epsilon_{i+1}^* (d_i - d_{i-1}) MK_4(\epsilon_{i+1}^*, \epsilon_{i+2}^*), \\ &\quad i = 1, 2, \dots, n-1 \end{aligned} \quad (21)$$

$$\begin{aligned} &MK_4(\epsilon_i^*, \epsilon_{i+1}^*) \\ &\equiv \epsilon_{i+1}^* MK_4(\epsilon_{i+1}^*, \epsilon_{i+2}^*), \\ &\quad i = 1, 2, \dots, n-1 \end{aligned} \quad (22)$$

$$\begin{aligned} &YK_3(\epsilon_i^*, \epsilon_{i+1}^*) \\ &\equiv \epsilon_i^* YK_3(\epsilon_{i-1}^*, \epsilon_i^*), \\ &\quad i = 1, 2, \dots, n-1 \end{aligned} \quad (23)$$

$$\begin{aligned} &YK_4(\epsilon_i^*, \epsilon_{i+1}^*, d_{i+1}, d_i) \\ &\equiv \epsilon_i^* (d_{i+1} - d_i) YK_3(\epsilon_{i-1}^*, \epsilon_i^*) \\ &\quad + \epsilon_{i+1}^* YK_4(\epsilon_{i-1}^*, \epsilon_i^*, d_i, d_{i-1}), \\ &\quad i = 1, 2, \dots, n-1 \end{aligned} \quad (24)$$

$$MK_3(\epsilon_n^*, \epsilon_{n+1}^*, d_n, d_{n-1}) \equiv d_n - d_{n-1} \quad MK_4(\epsilon_n^*, \epsilon_{n+1}^*) \equiv 1 \quad (25)$$

$$YK_3(\epsilon_0^*, \epsilon_1^*) \equiv 1 \quad YK_4(\epsilon_0^*, \epsilon_1^*, d_1, d_0) \equiv d_1 - d_0 \quad (26)$$

where, the symbols which have two definitions for $m \neq 0$; that is, Σ and γ_m in (4), A_m in (5), D_m in (8), $MK_1(\epsilon_n^*, \epsilon_{n+1}^*, d_n, d_{n-1}, \gamma_m)$ in (13), $MK_2(\epsilon_n^*, \epsilon_{n+1}^*, d_n, d_{n-1}, \gamma_m)$ in (14), $YK_1(\epsilon_0^*, \epsilon_1^*, d_1, d_0, \gamma_m)$ in (15), and $YK_2(\epsilon_0^*, \epsilon_1^*, d_1, d_0, \gamma_m)$ in (16) are selected for the eight cases as shown in Table I, respectively.

Using this general form, we can derive the Green's functions shown by Gish and Graham [2] and by Yamashita and Atsuki [3], [4].

Furthermore, we can easily show that these Green's functions satisfy the following reciprocity relation:

$$G(x_i, y_i; x_j, y_j) = G(x_j, y_j; x_i, y_i). \quad (27)$$

The approximate Green's function for a case of open microstrip line can be derived from the Green's function for case 1 by letting both $MK_1(n)$ and $MK_2(n)$ for case 1 in Table I be $\exp(-\gamma_m d_{n-1})$ and letting the sideward dimension a be large. Then, although the parameters for such a case can be obtained by the variational technique [2]–[4] using that Green's function, there is the undesirable property that the infinite series converge slowly when the sideward dimension is large.

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Comments on "Design Equations for an Interdigitated Directional Coupler"

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In the above short paper,¹ Ou derived some design equations for interdigital couplers. The purpose of this letter is to point out a limitation which may not be apparent upon reading the above work.

Because of Ou's simplifying assumptions, his method is not as accurate as he indicates. Let's consider the very popular –3-dB coupler example he selected. Turning to carefully plotted Bryant & Weiss data by Chambers [1, fig. 2], one can see that although S/H agree quite well, there is an obvious 30-percent disagreement in the W/H values of theory versus experiment [2], [3]. (Actually, Lange's data are very impressive—especially because he was announcing a new structure whose dimensions were intuitively and experimentally derived.)

Perhaps some will think that this letter is nit-picking. However, it can be shown by more rigorous theory than Ou used that "significant" strip-width errors can lead to couplers with poor isolation. Because the sources for poor isolation are many (uneven mode velocities in coupled microstrips, tolerances, connectors, wire bonds, etc.), the contributions of a linewidth error may not be obvious. If the primary error of the approximate method were in gap width, this letter would not be necessary because an error in coupling would be more apparent and the fix is obvious.

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Manuscript received December 24, 1975; revised March 16, 1976.

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¹ W. P. Ou, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 253–255, Feb. 1975.

Correction to "Analysis of Microstrip-Like Transmission Lines by Nonuniform Discretization of Integral Equations"

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In the above paper,¹ on page 195, the last line of the Abstract, and on page 198, line 4 of Section V, the abbreviation LSM was erroneously said to represent "linear synchronous motor." The correct meaning is "longitudinal-section magnetic modes."

Manuscript received May 1976.

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¹ E. Yamashita and K. Atsuki, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 195–200, Apr. 1976.